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Anti-de Sitter D-brane dynamics

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Abstract: We study the anti-de Sitter D-brane dynamics in hamiltonian formulation. By exploiting the existence of certain conserved charges, we solve the equation of motion for a D1-brane in the AdS_3 background and find the space-time dependent solutions. We further study the dynamics by mapping the problem to that of the open string tachyon, and examine the time dependent solutions in some detail.

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Contents

1. Introduction

Study of string theory in curved backgrounds is of great interest. Among these, Anti-de Sitter (AdS) spacetime has played a very important role in the evolution of string theory. The famous holographic AdS/CFT correspondence [1] relates quantum gravity in AdS space to a conformal field theory at the boundary. A concrete realization of this conjectured duality comes from the study of string theory on the $SL(2,\mathbb{R})$ Wess-Zumino-Witten model and its discrete orbifolds. The corresponding target space geometries are in general three dimensional AdS space with NS-NS three form flux. These backgrounds are considered to be the laboratory for testing the AdS/CFT duality beyond the supergravity approximation [2]. D-branes in these backgrounds are of interest and has been studied extensively in [3, 4].

The time dependent physics in string theory is equally challenging that includes many puzzling issues. Recently, a classical time dependent solution of the open string theory was suggested in [5] which describes the rolling of the tachyon in the valley of its potential on a unstable D-brane (or in brane-anti brane pair). The late time product of this 'rolling' has been interpreted as a classical 'tachyon matter'[6] state that is devoid of any obvious open string excitations, and which has the properties of the non-rotating, non-interacting, pressureless dust[7]. An important outcome of the recent observation of the tachyon condensation is that an effective action of the Dirac-Born-Infeld (DBI) type essentially captures all the intriguing aspects of the rolling tachyon solution of the full string theory. See [8] for a detailed review and a comprehensive list of references on the open string tachyon dynamics. More recently Kutasov [9, 10] gave a geometrical interpretation of the open string tachyon in the form of a rolling brane-the time dependent dynamics of the D-brane in the vicinity of a stack of NS5-branes, by using the DBI analysis. It was extended further into the full fledged string theory by constructing the relevant boundary states [11]. For more work along this direction see [12]. However most of the work has concentrated on the BPS D-brane falling into the NS5-brane and by showing the similarity between the radial mode with the perturbative tachyon on the unstable Dp-brane. But a similar study of the AdS D-brane dynamics is still lacking. In a recent paper [13] some properties of AdS D-branes has been outlined by using the DBI action on them. Hence a natural question arises whether one can study the evolution or the rolling of D-branes in AdS backgrounds by using some kind of 'tachyon mapping'. We address this problem along with finding out the solution to the equation of motion of the AdS D-branes in the hamiltonian formulation. The motivation for studying the present problem is manifold. Firstly, we extend the radion rolling dynamics beyond the flat space D-branes. We do this by using the hamiltonian formulation and by showing the existence of certain conserved charges. Second, we try to find out the solutions to the equations of motion for the AdS D-branes that can be interpreted as the D-brane moving towards the horizon of AdS spacetime.

The plan of the present paper is as follows. In section 2, we spell out the construction of the DBI action for a BPS D-brane in a general background in the presence of various worldvolume flux. In section 3 we focus our attention to the AdS D1-brane moving in the AdS_3 background towards the horizon in a controlled manner. By showing the invariance of the action under the existence of a particular symmetry (possibly broken at the lagrangian level), we are able to show two different conserved charges-corresponding to the total energy and the charge corresponding to the conserved dilatation. We further write down an expression for the conserved hamiltonian and find out the equations of motion for the D1-brane. Then we use the tachyon mapping to show the equivalence of the radial coordinate to that of rolling tachyon of the open string models, and in particular show that there are no plane wave solution for the geometrical tachyon near the horizon of the AdS_3 space. This is exactly the same behavior that the tachyon matter obey at the minimum of the potential, when the D-brane decays into the vacuum. We further map the solution to the global coordinate system of AdS_3 to study some general properties of the solution. In section 4, we present our conclusion.

2. Dirac-Born-Infeld action

We start with the Dirac-Born-Infeld action for Dp-brane in general background

$$S = -\tau_p \int d^{p+1}\xi e^{-\Phi} \sqrt{-\det \mathbf{A}_{\mu\nu}} \,, \qquad \mathbf{A}_{\mu\nu} = \gamma_{\mu\nu} + F_{\mu\nu} \,, \tag{2.1}$$

where τ_p is Dp-brane tension, $\Phi(X)$ is the dilaton. The $\gamma_{\mu\nu}\,, \mu, \nu = 0, \ldots, p$ is embedding of the metric into the Dp-brane worldvolume

$$\gamma_{\mu\nu} = g_{MN}\partial_\mu X^M \partial_\nu X^N \,, \qquad M, N = 0, \ldots, D \,. \tag{2.2}$$

In (2.1) the induced two-form $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = b_{MN}\partial_\mu X^M \partial_\nu X^N + \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{2.3}$$

Thanks to the diffeomorphism invariance of the worldvolume theory, it is natural to fix some of the spacetime coordinates X^M to be equal to the worldvolume coordinates ξ^μ:

$$X^\mu = \xi^\mu \,, \qquad \mu = 0, \dots, p \,. \tag{2.4}$$

Note that this choice generally leads to the induced metric $\gamma_{\mu\nu}$ and two form $F_{\mu\nu}$ to be functions of ξ and X^I where $I, J, \dots = p+1, \dots, D$ label coordinates that are transverse to the worldvolume of Dp-brane:

$$\begin{aligned}\gamma_{\mu\nu} &= g_{\mu\nu}(\xi, X) + g_{\mu I}(\xi, X)\partial_\nu X^I + g_{J\nu}(\xi, X)\partial_\mu X^J + g_{IJ}(\xi, X)\partial_\mu X^I \partial_\nu X^J \,, \\ F_{\mu\nu} &= b_{\mu\nu}(\xi, X) + b_{\mu I}(\xi, X)\partial_\nu X^I + b_{J\nu}(\xi, X)\partial_\mu X^J + b_{IJ}(\xi, X)\partial_\mu X^I \partial_\nu X^J + \\ &\quad + \partial_\mu A_\nu - \partial_\mu A_\nu \,. \end{aligned} \tag{2.5}$$

In the following we will consider the case where the metric is diagonal so that $g_{\mu I} = g_{J\nu} = 0$. Now the equation of motion for X^I derived from (2.1) takes the form

$$\begin{aligned}&\frac{\delta e^{-\Phi}}{\delta X^I}\sqrt{-\det \mathbf{A}} + \frac{e^{-\Phi}}{2}\left[\frac{\delta g_{\mu\nu}}{\delta X^I} + \frac{\delta g_{KL}}{\delta X^I}\partial_\mu X^K \partial_\nu X^L + \frac{\delta B_{\mu\nu}}{\delta X^I}\right]\left(\mathbf{A}^{-1}\right)^{\nu\mu}\sqrt{-\det \mathbf{A}} - \\ &\qquad - \frac{1}{2}\partial_\mu\left[e^{-\Phi} g_{IK}\partial_\nu X^K\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} + \left(\mathbf{A}^{-1}\right)^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] - \\ &\qquad - \frac{1}{2}\partial_\mu\left[e^{-\Phi} b_{I\nu}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} - \left(\mathbf{A}^{-1}\right)^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] - \\ &\qquad - \frac{1}{2}\partial_\mu\left[e^{-\Phi} b_{IK}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} - \left(\mathbf{A}^{-1}\right)^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] = 0 \,. \end{aligned} \tag{2.6}$$

Finally, we should also determine the equation of motion for the gauge field A_μ:

$$\partial_\nu\left[e^{-\Phi}\left(\left(\mathbf{A}^{-1}\right)^{\mu\nu} - \left(\mathbf{A}^{-1}\right)^{\nu\mu}\right)\sqrt{-\det \mathbf{A}}\right] = 0 \,. \tag{2.7}$$

In what follows we restrict ourselves to the examples when the induced metric does not depend on the worldvolume coordinates ξ^μ. More precisely, we also presume that the metric and the two form field depends only on a single coordinate, say R which is the reflection of the rotation symmetry of the background.

3. D-brane dynamics in AdS$_3$-background

AdS$_3$ backgrounds offer a laboratory for testing the gauge-gravity duality beyond the supergravity approximation. Our aim in this section is to study the time dependent dynamics of the AdS D-branes. We do this by applying the most general procedure depicted in the previous section. We concentrate mostly on the particular case of D1-brane in AdS_3 background. The metric and the NS-NS two form field of such a background is given by

$$ds^2 = -r^2 dt^2 + r^2 dz^2 + r^{-2} dr^2 \,, \qquad b = r^2 dt \wedge dz \,. \tag{3.1}$$

In this particular parametrization, the horizon corresponds to $r \to 0$. Let us consider the D1-brane that is stretched in (t, z)-plane. In this case the only embedding coordinate is R that we presume depends both on time and z

$$\mathrm{R} = \mathrm{R}(z, t) \,. \tag{3.2}$$

We also presume that the gauge potential has a nonzero component A_z only. With this ansatz the matrix $\mathbf{A}_{\mu\nu}$ introduced in (2.1) takes the form

$$\mathbf{A}_{\mu\nu} = \begin{pmatrix} -\mathrm{R}^2 + \frac{(\partial_0 \mathrm{R})^2}{\mathrm{R}^2} & \frac{\partial_0 \mathrm{R} \partial_z \mathrm{R}}{\mathrm{R}^2} + \mathrm{R}^2 + \partial_0 A_z \\ \frac{\partial_0 \mathrm{R} \partial_z \mathrm{R}}{\mathrm{R}^2} - \mathrm{R}^2 - \partial_0 A_z & \mathrm{R}^2 + \frac{(\partial_z \mathrm{R})^2}{\mathrm{R}^2} \end{pmatrix} \tag{3.3}$$

and hence the action takes the following form

$$S = -\tau_2 \int dtdz \sqrt{\mathrm{R}^4 + (\partial_z \mathrm{R})^2 - (\partial_0 \mathrm{R})^2 - (\mathrm{R}^2 + \partial_0 A_z)^2} \, . \tag{3.4}$$

Observe that under the transformations

$$\mathrm{R}'(t', z') = \lambda \mathrm{R}(t, z) \, , \quad A'_z(t', z') = \lambda A(t, z) \, , \quad t' = \lambda^{-1} t \, , \quad z' = \lambda^{-1} z \, , \quad \lambda = \mathrm{const.} \tag{3.5}$$

the action above is invariant in the following sense

$$\int dt' dz' \mathcal{L}(\mathrm{R}', A'_z, t', z') = \int dtdz \mathcal{L}(\mathrm{R}, A_z, t, z) \, . \tag{3.6}$$

Using this fact, we should be able to find two conserved charges, corresponding to the conserved energy $\mathcal{E}$ and the charge corresponding to the conserved dilatation.

3.1 Conserved charges

Adopting the standard procedure we get the following form of the world volume stress energy tensor $\Theta_{\mu\nu}$

$$\begin{aligned} \Theta^\mu_\nu &= -\mathcal{L}\delta^\mu_\nu - \frac{\eta^{\mu\kappa} \partial_\kappa \mathrm{R} \partial_\nu \mathrm{R}}{\sqrt{\mathrm{R}^4 + (\partial_z \mathrm{R})^2 - (\partial_0 \mathrm{R})^2 - (\mathrm{R}^2 + \partial_0 A_z)^2}} + \\ &+ \frac{\delta^\mu_0 (\mathrm{R}^2 + \partial_0 A_z)}{\sqrt{\mathrm{R}^4 + (\partial_z \mathrm{R})^2 - (\partial_0 \mathrm{R})^2 - (\mathrm{R}^2 + \partial_0 A_z)^2}} \partial_\nu A_z \, . \end{aligned} \tag{3.7}$$

Note that the trace of the stress energy tensor does not vanish

$$\Theta^\mu_\mu = \frac{\eta^{\mu\nu} \partial_\mu \mathrm{R} \partial_\nu \mathrm{R} - (\mathrm{R}^2 + \partial_0 A_z) \partial_0 A_z}{\sqrt{\mathrm{R}^4 + (\partial_z \mathrm{R})^2 - (\partial_0 \mathrm{R})^2 - (\mathrm{R}^2 + \partial_0 A_z)^2}} \, , \tag{3.8}$$

that implies that the lagrangian given above does not define conformal field theory. In spite of the fact we can find the dilatation like charge that is a generator of the transformations given in (3.5) that for small λ can be written as

$$x'^\mu = (1+\epsilon)^{-1} x^\mu = x^\mu - \epsilon x^\mu \tag{3.9}$$

$$\mathrm{R}'(\mu') = (1+\epsilon) \mathrm{R}(x^\mu) = \mathrm{R}(x^\mu) + \epsilon \mathrm{R}(x^\mu) \, , \tag{3.10}$$

$$A'_z(\mu') = (1+\epsilon) A_z(x^\mu) = A_z(x^\mu) + \epsilon A_z(x^\mu) \tag{3.11}$$

and hence we obtain following form of the dilatation current j^μ_D:

$$\begin{aligned} j^\mu_D &= \left(-\mathcal{L}\delta^\mu_\nu + \frac{\delta \mathcal{L}}{\delta \partial_\mu \mathrm{R}} \partial_\nu \mathrm{R} + \frac{\delta \mathcal{L}}{\delta \partial_\mu A_z} \partial_\nu A_z \right) (-x^\nu) - \frac{\delta \mathcal{L}}{\delta \partial_\mu \mathrm{R}} \mathrm{R} - \frac{\delta \mathcal{L}}{\delta \partial_\mu A_z} A_z \, , \\ j^0_D(z) &= -t\mathcal{H}(z) - z \left(\Pi_\mathrm{R}(z) \partial_z \mathrm{R}(z) + \Pi_A(z) \partial_z A_z(z) \right) - \Pi_\mathrm{R}(z) \mathrm{R} - \Pi_A(z) A_z \, , \end{aligned} \tag{3.12}$$

where $\mathcal{H}$ is hamiltonian density and where $\Pi_R(z)\,,\Pi_A(z)$ are the momenta conjugate to R and A_z:

$$\Pi_R(z)=\frac{\delta\mathcal{L}}{\delta\partial_0 R(z)}\,,\Pi_A(z)=\frac{\delta\mathcal{L}}{\delta\partial_0 A_z(z)}\,. \tag{3.13}$$

Note that $j^\mu_D\neq x^\nu\theta^\mu_\nu$ that again shows that this two dimensional field theory is not the conformal one.

From (3.12) we obtain the following conserved charge

$$Q_D(t)=\int_{-\infty}^{\infty}dz j^0_D(z)=-tH-\int_{-\infty}^{\infty}dz\left(z\left[\Pi_{\mathrm{R}}\partial_z\mathrm{R}+\Pi_A\partial_z A_z\right]+\Pi_{\mathrm{R}}\mathrm{R}+\Pi_A A_z\right). \tag{3.14}$$

As a check, the Poisson bracket of Q_D with R gives

$$\begin{aligned}\{Q_D,\mathrm{R}(z)\}&=-t\{H,\mathrm{R}(z)\}-\int dz'\left(z'\partial'_z\mathrm{R}\{\Pi_{\mathrm{R}}(z'),\mathrm{R}(z)\}+\mathrm{R}(z')\{\Pi_{\mathrm{R}}(z'),\mathrm{R}(z)\}\right)\\&=-t\partial_0\mathrm{R}(z)-z\partial_z\mathrm{R}(z)-\mathrm{R}(z)\,,\end{aligned} \tag{3.15}$$

using the canonical relations

$$\left\{\Pi_{\mathrm{R}}(z),\mathrm{R}(z')\right\}=\delta(z-z')\,,\qquad\left\{\Pi_A(z),A_z(z')\right\}=\delta(z-z') \tag{3.16}$$

and the equation of motions

$$\begin{aligned}\partial_0\mathrm{R}(z)&=\{H,\mathrm{R}(z)\}\,, &\quad \partial_0\Pi_{\mathrm{R}}(z)&=\{H,\Pi_{\mathrm{R}}(z)\}\,,\\ \partial_0 A_z(z)&=\{H,A_z(z)\}\,, &\quad \partial_0\Pi_{\mathrm{A}}(z)&=\{H,\Pi_{\mathrm{A}}(z)\}\,.\end{aligned} \tag{3.17}$$

In the same way we could study the Poisson bracket of Q_D with A_z.

For later purposes we now determine an explicit form of the hamiltonian density. To do this let us consider the lagrangian density in the following form

$$\mathcal{L}=-\sqrt{V-\sum_i(f_i(\partial_0\Phi^i)^2+B_i\partial_0\Phi^i)}\equiv-\triangle\,, \tag{3.18}$$

where V contain scalar potential for various fields Φ^i and also the spatial gradients of these fields, and f_i and B_i are constants. The conjugate momentum P_i to Φ^i takes the form

$$P_i=\frac{2f_i\partial_0\Phi^i+B_i}{2\triangle}\,,\qquad\partial_0\Phi^i=\frac{1}{2f_i}\left(2P_i\triangle-B_i\right) \tag{3.19}$$

so that the hamiltonian density takes the form

$$\mathcal{H}=\sqrt{\left(V+\sum_i\frac{B_i^2}{4f_i}\right)\left(1+\sum_i\frac{P_i^2}{f_i}\right)}-\sum_i\frac{B_iP_i}{2f_i}\,. \tag{3.20}$$

In case of D1-brane in AdS_3 background the action was given in (3.4) so that we get

$$V=(\partial_z\mathrm{R})^2\,,\qquad f_{\mathrm{R}}=1\,,\qquad f_{A_z}=1\,,\qquad B_{A_z}=2\mathrm{R}^2 \tag{3.21}$$

and hence the hamiltonian density (3.20) is equal to

$$\mathcal{H} = \sqrt{((\partial_z \mathrm{R})^2 + \mathrm{R}^4)(1 + \Pi_A^2 + \Pi_\mathrm{R}^2)} - \mathrm{R}^2 \Pi_A \equiv \sqrt{\mathcal{K}} - \mathrm{R}^2 \Pi_A \,. \tag{3.22}$$

It is also instructive to calculate the Poisson bracket of Q_D with H. This can be easily performed using the canonical relations (3.16) and the equation of motions (3.17)

$$\begin{aligned}\{Q_D, H\} = \partial_0 \Bigg[\int_{-\infty}^{\infty} dz \Big[z\, (\Pi_\mathrm{R}(z)\partial_z \mathrm{R}(z) + \Pi_A(z)\partial_z A_z(z)) + \\ + \Pi_\mathrm{R}(z)\mathrm{R}(z) + \Pi_A(z) A_z(z)\Big]\Bigg] = \frac{dQ}{dt} + H \,,\end{aligned} \tag{3.23}$$

using

$$\{A(z)B(z), H\} = \{A(z), H\}\, B(z) + A(z)\, \{B(z), H\} \tag{3.24}$$

and also the fact that

$$\begin{aligned}\frac{dQ_D}{dt} = -H + \partial_0 \Bigg(\int_{-\infty}^{\infty} dz \Big[z\, (\Pi_\mathrm{R}(z)\partial_z \mathrm{R}(z) + \Pi_A(z)\partial_z A_z(z)) + \\ + \Pi_\mathrm{R}(z)\mathrm{R}(z) + \Pi_A(z) A_z(z)\Big]\Bigg) \,.\end{aligned} \tag{3.25}$$

Note that (3.23) is in agreement with the definition of the time evolution of Q_D

$$\frac{dQ_D}{dt} = \frac{\partial Q_D}{\partial t} + \{H, Q_D\} \,. \tag{3.26}$$

Let us now study in more details the canonical equation of motion. Firstly, since the hamiltonian does not explicitly depend on A_z it follows that Π_A is conserved

$$\partial_0 \Pi_A(z) = \{H, \Pi_A(z)\} = 0 \,. \tag{3.27}$$

We can consistently solve the equation of motion with the ansatz that $\Pi_A = \text{const.}$ with the physical interpretation that Π_A gives the density of fundamental strings.

Using (3.22) the equation of motion for Π_R and R are equal to

$$\begin{aligned}\partial_0 \mathrm{R}(z) &= \frac{\delta H}{\delta \Pi_\mathrm{R}(z)} = \frac{((\partial_z \mathrm{R})^2 + \mathrm{R}^4)\Pi_\mathrm{R}}{\sqrt{\mathcal{K}}} \,, \\ \partial_0 \Pi_\mathrm{R}(z) &= -\frac{\delta H}{\delta \mathrm{R}(z)} = -\frac{2\mathrm{R}^3(1 + \Pi_A^2 + \Pi_\mathrm{R}^2)}{\sqrt{\mathcal{K}}} + 2\Pi_A \mathrm{R} + \partial_z \left[\frac{\partial_z \mathrm{R}(1 + \Pi_A^2 + \Pi_\mathrm{R}^2)}{\sqrt{\mathcal{K}}}\right] , \\ \partial_0 A_z(z) &= \frac{\delta H}{\delta \Pi_A} = \frac{(\partial_z \mathrm{R})^2 + \mathrm{R}^4)\Pi_A}{\sqrt{\mathcal{K}}} - \mathrm{R}^2 \,.\end{aligned} \tag{3.28}$$

Our goal is to find the solution of the canonical equations of motion that describes the time evolution of an inhomogenous D1-brane. The structure of the hamiltonian density (3.22) suggests that we should search for the solution of R of the form

$$\partial_z \mathrm{R} = k_z \mathrm{R}^2 \,. \tag{3.29}$$

A general solution of R can be given by (with an integration constant $f(t)$)

$$\frac{1}{\mathrm{R(z,t)}} = -(k_z z + f(t))\,, \tag{3.30}$$

and we also get

$$\mathcal{H} = \mathrm{R}^2\left(\sqrt{(1+k_z^2)(1+\Pi_A^2+\Pi_\mathrm{R}^2)} - \Pi_A\right). \tag{3.31}$$

Using (3.31) we express Π_R as function of $\mathcal{H}$ and R

$$\Pi_\mathrm{R} = \pm\frac{1}{\mathrm{R}^2\sqrt{1+k_z^2}}\sqrt{(\mathcal{H}+\Pi_A\mathrm{R}^2)^2 - \mathrm{R}^4(1+k_z^2)(1+\Pi_A^2)} \tag{3.32}$$

and insert it to the first equation in (3.28)

$$\partial_0\mathrm{R} = \pm\sqrt{1+k_z^2}\mathrm{R}^2\sqrt{1-\frac{\mathrm{R}^4(1+\Pi_A^2)(1+k_z^2)}{(\mathcal{H}+\Pi_A\mathrm{R}^2)^2}}\,. \tag{3.33}$$

If we also use the fact that

$$\partial_0\mathrm{R} = -\dot{f}\mathrm{R}^2 \tag{3.34}$$

the equation (3.33) simplifies as

$$\dot{f} = \mp\sqrt{1+k_z^2}\sqrt{1-\frac{\mathrm{R}^4(1+\Pi_A^2)(1+k_z^2)}{(\mathcal{H}+\Pi_A\mathrm{R}^2)^2}}\,. \tag{3.35}$$

Let us insert (3.31) into the equation above and we get

$$\dot{f} = \mp\sqrt{1+k_z^2}\sqrt{1-\frac{1+\Pi_A^2}{(1+\Pi_A^2+\Pi_\mathrm{R}^2)}}\,. \tag{3.36}$$

By comparing the left and the right side of this equation we deduce that Π_R depends on t only. Then the equation of motion for Π_R simplifies as

$$\partial_0\Pi_\mathrm{R} - \frac{2\mathrm{R}}{\sqrt{1+k_z^2}}\sqrt{1+\Pi_A^2+\Pi_R^2} + 2\Pi_A\mathrm{R}\,. \tag{3.37}$$

Moreover, looking at the form of the time and spatial dependence of R given in (3.30), one expects that $f(t)$ should be linear function of t as well. In fact, the worldvolume theory is still Lorentz invariant. With this presumption $\dot{f} = \text{const}$, and hence the equation (3.36) implies that $\Pi_\mathrm{R} = \text{const.}$ as well. Now the equation (3.37) implies

$$\Pi_\mathrm{R}^2 = \Pi_A^2 k_z^2 - 1 \tag{3.38}$$

and from (3.36) we get

$$f = \mp t\sqrt{k_z^2 - \frac{1}{\Pi_A^2}} + f_0 = k_0 t + f_0\,, \qquad f_0 = \text{const.}\,, \tag{3.39}$$

where we have introduced the following notation

$$k_0 = \mp\sqrt{k_z^2 - \frac{1}{\Pi_A^2}}\,, \qquad -k_0^2 + k_z^2 = -\frac{1}{\Pi_A^2}\,. \tag{3.40}$$

With the above, the general solution of R in (3.30) can be written as

$$-\frac{1}{\mathrm{R}(t,z)} = k_0 t + k_z z + f_0\,. \tag{3.41}$$

In summary we have found time and space dependent solution of D1-brane equation of motion using the hamiltonian formalism. We will say more words about this solution in the next section, where we will study the same problem with the help of the lagrangian density for the new mode, "geometrical tachyon" that we define below.

3.2 Tachyon mapping

In this section, we try to find out the solutions to the equation of motion for the radial coordinate ('radion') on the D1-brane, by mapping it to the open string tachyon on the unstable branes. Our strategy will be as follows. We start with the action for a D1-brane in the AdS_3 background that has already been derived in the previous section

$$S = -\tau_2 \int d^2x \sqrt{\mathrm{R}^4 + (\partial_1\mathrm{R})^2 - (\partial_0\mathrm{R})^2 - (\mathrm{R}^2 + \partial_0 A_1)^2}\,, \tag{3.42}$$

where we have replaced $\partial_z\mathrm{R} \to \partial_1\mathrm{R}$, and $A_z \to A_1$ as compared to the previous section. Now let us map the above action to the tachyon effective action of the open string model, with the geometric tachyon $\mathcal{T}$ being the only dynamical field involved. As opposite to the standard construction where there is no gauge field present, here the situation seems to be more complicated. We propose the following procedure. First we pass to the hamiltonian formalism. This was done in the previous section with the hamiltonian density given by

$$\mathcal{H} = \sqrt{((\partial_1\mathrm{R})^2 + \mathrm{R}^4)(1 + \Pi_A^2 + \Pi_R^2)} - \mathrm{R}^2\Pi_A \equiv \sqrt{\mathcal{K}} - \mathrm{R}^2\Pi_A\,. \tag{3.43}$$

As we have shown in the previous section, the conjugate momentum Π_A is a constant. Now we see that the dynamical modes in (3.43) are Π_R and R. Then we perform an inverse Legendre transformation to get the lagrangian for R that depends on the constant Π_A as well. More precisely, we define the new lagrangian density for R as

$$\mathcal{L}_{\mathrm{new}} = \Pi_{\mathrm{R}}\partial_0\mathrm{R} - \mathcal{H}\,. \tag{3.44}$$

Using the fact that

$$\partial_0\mathrm{R} = \frac{((\partial_1\mathrm{R})^2 + \mathrm{R}^4)\Pi_{\mathrm{R}}}{\sqrt{\mathcal{K}}} \tag{3.45}$$

we get

$$\mathcal{L}_{\mathrm{new}} = -\frac{((\partial_1\mathrm{R})^2 + \mathrm{R}^4)(1 + \Pi_A^2)}{\sqrt{\mathcal{K}}} + \mathrm{R}^2\Pi_A\,. \tag{3.46}$$

As a next step, we express $\mathcal{K}$ that depends on Π_R and R, as functions of R and ∂_0R. Using (3.45) we obtain

$$\Pi_{\mathrm{R}} = \frac{\partial_0 \mathrm{R}\sqrt{1+\Pi_A^2}}{\sqrt{((\partial_1 \mathrm{R})^2 + \mathrm{R}^4) - (\partial_0 \mathrm{R})^2}} \tag{3.47}$$

and hence

$$\mathcal{K} = \frac{((\partial_1 \mathrm{R})^2 + \mathrm{R}^4)^2(1+\Pi_A^2)}{(\partial_1 \mathrm{R})^2 + \mathrm{R}^4 - (\partial_0 \mathrm{R})^2} \,. \tag{3.48}$$

Finally we obtain the lagrangian density $\mathcal{L}_{\mathrm{new}}$ in the form

$$\mathcal{L}_{\mathrm{new}} = -\sqrt{1+\Pi_A^2}\mathrm{R}^2\sqrt{1+\frac{1}{\mathrm{R}^4}((\partial_1 \mathrm{R})^2 - (\partial_0 \mathrm{R})^2)} + \Pi_A \mathrm{R}^2 \,. \tag{3.49}$$

Using the manifestly covariant lagrangian density (3.49) we can introduce the so called 'geometrical tachyon' ($\mathcal{T}$) in the following form

$$\frac{d\mathrm{R}}{\mathrm{R}^2} = d\mathcal{T} \,, \qquad \mathcal{T} = -\frac{1}{\mathrm{R}}. \tag{3.50}$$

Now

$$\mathrm{R} \to 0 \Longrightarrow \mathcal{T} \to -\infty \,, \qquad \mathrm{R} \to \infty \Longrightarrow \mathcal{T} \to 0 \,. \tag{3.51}$$

With the identification (3.50), the lagrangian (3.49) takes the form

$$\mathcal{L} = -\sqrt{1+\Pi_A^2}\frac{1}{\mathcal{T}^2}\sqrt{1+\eta^{\mu\nu}\partial_\mu \mathcal{T}\partial_\nu \mathcal{T}} + \Pi_A \frac{1}{\mathcal{T}^2} \,. \tag{3.52}$$

Our goal now is to solve the equation of motion for $\mathcal{T}$ that is derived from (3.52)

$$\frac{2\sqrt{1+\Pi_A^2}}{\mathcal{T}^3}\frac{1}{\sqrt{1+\eta^{\mu\nu}\partial_\mu \mathcal{T}\partial_\nu \mathcal{T}}} + \frac{\sqrt{1+\Pi_A^2}}{\mathcal{T}^2}\partial_\mu\left[\frac{\eta^{\mu\nu}\partial_\nu \mathcal{T}}{\sqrt{1+\eta^{\mu\nu}\partial_\mu \mathcal{T}\partial_\nu \mathcal{T}}}\right] - \frac{2\Pi_A}{\mathcal{T}^3} = 0 \,. \tag{3.53}$$

Let us presume that there exists a solution, to the equation of motion (3.53), of the following the form

$$\eta^{\mu\nu}\partial_\mu \mathcal{T}\partial_\nu \mathcal{T} = \mathrm{K}^2 \,, \qquad \eta^{\mu\nu}\partial_\mu\partial_\nu \mathcal{T} = 0 \,, \tag{3.54}$$

with a constant K to be determined. For such an ansatz (3.53) reduces into

$$\frac{1}{\mathcal{T}^3}\left(\frac{\sqrt{1+\Pi_A^2}}{\sqrt{1+\mathrm{K}^2}} - \Pi_A\right) = 0 \,. \tag{3.55}$$

The above equation determines the constant K in terms of Π_A as

$$\mathrm{K}^2 = \frac{1}{\Pi_A^2} \,. \tag{3.56}$$

Let us now try to solve the equation (3.54) with a plane wave ansatz

$$\mathcal{T} = Ae^{k_\mu x^\mu} \,. \tag{3.57}$$

For (3.57) the equations (3.54) take the form

$$k_0^2 - k_1^2 = 0\,, \qquad -k_0^2 + k_1^2 = \mathrm{K}^2 \tag{3.58}$$

that however implies $\mathrm{K} = 0$. But that is not consistent with (3.56) where we presume a finite Π_A. This indicates that for the geometrical tachyon, there exists no plane wave solution at the horizon of the AdS space. Infact, we can note that as the D1-brane rolls towards the horizon, there is no physical open string excitations left, a fact that is consistent with the observation [7]

Instead of using the plane-wave ansatz (3.57) let us consider the linear ansatz

$$\mathcal{T} = k_\mu x^\mu + B\,, \tag{3.59}$$

with a constant B. Now, the first condition in (3.54) takes the form

$$\eta^{\mu\nu} k_\mu k_\nu = \mathrm{K}^2\,, \tag{3.60}$$

while the second condition $\eta^{\mu\nu}\partial_\mu\partial_\nu\mathcal{T} = 0$ is satisfied trivially. Now using (3.56) we get

$$k_0^2 = k_1^2 - \mathrm{K}^2 = k_1^2 - \frac{1}{\Pi_A^2}\,. \tag{3.61}$$

In terms of the original variable R we then get the solution

$$-\frac{1}{\mathrm{R}} = k_0 x^0 + k_1 x^1 + b\,, \qquad b = \mathrm{const}\,. \tag{3.62}$$

We see that this solution coincides exactly with the solution (3.41) derived in the previous section in the hamiltonian formalism.

We can check the consistency of this solution by considering the case when $k_0 = 0$. In this case (from (3.61)) we get

$$k_1^2 = \frac{1}{\Pi_A^2} \tag{3.63}$$

and hence

$$\frac{1}{\mathrm{R}} = -k_1 x^1 + b\,. \tag{3.64}$$

First of all, the fact that R should be positive implies that we should take $k_1 = -\frac{1}{|\Pi_A|}$. Now we determine the constant b from the requirement that for $x^1 \to x_0^1$, R blows up. Hence we get

$$b = -\frac{1}{|\Pi_A|} x_0^1\,. \tag{3.65}$$

Taking the above ingredients, we obtain the following solution for R that is spatial dependent

$$\mathrm{R} = \frac{|\Pi_A|}{x^1 - x_0^1} \tag{3.66}$$

that coincides with the solution determined in [3].

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Let us return to the time and space dependent solution for R. We see from (3.62) that the point where the D-brane is stretched to $\mathrm{R} = \infty$ is time dependent with the time dependence is governed by the equation

$$0 = k_1 x^1 + k_0 x^0 + b\,. \tag{3.67}$$

To study the general properties of the solution (3.62), we map this to the global coordinates of the AdS_3. Recall that in global coordinates AdS_3 background metric and the the three-form field strength takes the form

$$ds^2 = L^2[-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\phi^2] \tag{3.68}$$

and

$$H = dB = L^2 \sinh(2\rho) d\rho \wedge d\phi \wedge dt\,, \qquad B = L^2 \sinh^2\rho d\phi \wedge d\tau\,. \tag{3.69}$$

Poincare and global coordinates are related through the relation

$$x^1 \pm x^0 = \frac{1}{r}(\sinh\rho \sin\phi \pm \cosh\rho \sin\tau) \tag{3.70}$$

and

$$r = \cosh\rho\cos\tau + \sinh\rho\cos\phi\,. \tag{3.71}$$

Now it is easy to see that in global coordinates the solution (3.62) takes the form

$$(k_1 + b)\sinh\rho\sin\phi + (k_0 + b)\cosh\rho\sin\tau = -1\,. \tag{3.72}$$

This has the form of the solution given in [3]. To see this more clearly recall that in the static case $k_0 = 0$ the upper equation takes the form

$$-\frac{1}{|\Pi_A|}(1 + x_0^1)\sinh\rho\sin\phi - \frac{1}{|\Pi_A|}x_0^1\cosh\rho\sin\tau = -1\,. \tag{3.73}$$

This is in agreement with the result given in [3] where the solution with the boundary condition $x_0^1 = 0$ was given.

3.3 More observation

Let us now study the embedding equation for given D1-brane. To find such an equation we should find the relation between the variables z, r, t and the coordinates X^0, X^1, X^2, X^3 that define AdS_3 as embedded hypersurface in the four dimensional space through the equation

$$-(X^0)^2 + (X^1)^2 + (X^2)^2 - (X^3)^2 = -1\,. \tag{3.74}$$

The relation between the Poincare and the cartesian coordinates takes the form

$$X^0 + X^1 = r\,, \qquad X^2 \pm X^3 = r w^{\pm}\,, \qquad X^0 - X^1 = \frac{1}{r} + r w^+ w^-\,, \tag{3.75}$$

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where $w^{\pm} = x^1 \pm x^0$. Alternatively

$$\begin{aligned} X^2 &= rx^1 , \qquad X^3 = rx^0 , \\ X^0 &= \frac{1}{2}\left(r + \frac{1}{r} + r\left((x^1)^2 - (x^0)^2\right)\right) , \\ X^1 &= \frac{1}{2}\left(r - \frac{1}{r} - r\left((x^1)^2 - (x^0)^2\right)\right) . \end{aligned} \tag{3.76}$$

Since we know that the static D1-brane in AdS_3 is described by the solution $r = \frac{|\Pi_A|}{x^1}$, we obtain from the first equation in (3.76) that in global coordinates corresponds to $X^2 = |\Pi_A|$. If we insert this relation into (3.74) we get

$$(X^0)^2 + (X^3)^2 - (X^1)^2 = 1 + \Pi_A^2 . \tag{3.77}$$

The above defines AdS_2 embedded hypersurface in a three dimensional space.

For general solution (3.62) we can write (for $b = 0$)

$$k_1 X^2 + k_0 X^3 = r(k_1 x^1 + k_0 x^0) = -1 \tag{3.78}$$

Then we get

$$X^2 = -\frac{1}{k_1}(1 + k_0 X^3) \tag{3.79}$$

and hence the embedding equation takes the form

$$(X^0)^2 - (X^1)^2 + (1 - \frac{k_0^2}{k_1^2})\left(X^3 - \frac{k_0}{k_1^2(1 - \frac{k_0^2}{k_1^2})}\right)^2 = 1 + \frac{1}{k_1^2} + \frac{k_0^2}{k_1^4(1 - \frac{k_0^2}{k_1^2})} = 1 + \Pi_A^2 , \tag{3.80}$$

where we have used from eqn. (3.61) that $\frac{1}{\Pi_A^2} = k_1^2 - k_0^2$. One can check once again that the above defines AdS_2 D-brane, by comparing with (3.77). More precisely, if we define

$$\tilde{X}^0 = X^0 , \qquad \tilde{X}^1 = X^1 , \qquad \tilde{X}^3 = X^3 - \frac{k_0}{k_1^2(1 - \frac{k_0^2}{k_1^2})} \tag{3.81}$$

and then perform the following rescaling

$$(1 - \frac{k_0^2}{k_1^2})(\tilde{X}^3)^2 \rightarrow (\tilde{X}^3)^2 , \tag{3.82}$$

then we see that (3.81) coincides with (3.74).

4. Conclusion

In this paper, we have studied dynamics of the Anti-de Sitter D-branes, by using the hamiltonian formulation. By exploiting the existence of conserved charges, we determine the solution that is both space and time dependent to the D1-brane equation of motion. We further use the tachyon mapping to relate the radial mode on the D1-brane to the open

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string tachyon. We write down explicitly the lagrangian and the equation of motion in terms of the new geometrical tachyon field. By solving the equations of motion, we have showed the absence of plane-wave solutions. Infact a linear ansatz does solve the equations of motion for the geometrical tachyon field, and that is indeed consistent with the solution predicted by directly solving the hamiltonian equations. This solution further reduces to the static solution of the AdS-branes that has been discussed in the literature earlier. We have also showed that the D1-brane discussed in this paper, is indeed the AdS_2 brane. The analysis done in this paper can easily be generalized to the higher AdS spaces in order to study the dynamics of all the AdS D-brane. For example, one can study the time evolution of the D2-brane in $\mathrm{AdS}_3\times \mathrm{S}^3$ background, and study the possible stable tubular solutions. It is also interesting to examine the AdS-brane dynamics in other curved backgrounds like the NS5-brane and the macroscopic fundamental string backgrounds. It is worthy examining the brane dynamics from the gauge theory view point, by using the gauge-gravity duality. Another interesting problem would be to study the brane dynamics from the full conformal field theory view point, namely by constructing the relevant boundary states for these branes, and studying their properties.

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References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] A. Giveon, D. Kutasov and N. Seiberg, *Comments on string theory on AdS_3*, *Adv. Theor. Math. Phys.* **2** (1998) 733 [hep-th/9806194];
J. de Boer, H. Ooguri, H. Robins and J. Tannenhauser, *String theory on AdS_3*, *JHEP* **12** (1998) 026 [hep-th/9812046];
D. Kutasov and N. Seiberg, *More comments on string theory on AdS_3*, *JHEP* **04** (1999) 008 [hep-th/9903219];
J.M. Maldacena and H. Ooguri, *Strings in AdS_3 and $\mathrm{SL}(2,\mathbb{R})$ WZW model, I*, *J. Math. Phys.* **42** (2001) 2929 [hep-th/0001053];
Strings in AdS_3 and the $\mathrm{SL}(2,\mathbb{R})$ WZW model, III. Correlation functions, *Phys. Rev.* **D 65** (2002) 106006 [hep-th/0111180];
J.M. Maldacena, H. Ooguri and J. Son, *Strings in AdS_3 and the $\mathrm{SL}(2,\mathbb{R})$ WZW model, II. Euclidean black hole*, *J. Math. Phys.* **42** (2001) 2961 [hep-th/0005183].

[3] C. Bachas and M. Petropoulos, *Anti-de-sitter D-branes*, *JHEP* **02** (2001) 025 [hep-th/0012234].

[4] P.M. Petropoulos and S. Ribault, *Some remarks on anti-de Sitter D-branes*, *JHEP* **07** (2001) 036 [hep-th/0105252];
A. Giveon, D. Kutasov and A. Schwimmer, *Comments on D-branes in AdS_3*, *Nucl. Phys.* **B 615** (2001) 133 [hep-th/0106005];
C. Bachas, *D-branes in some near-horizon geometries*, hep-th/0106234;
Y. Hikida and Y. Sugawara, *Boundary states of D-branes in AdS_3 based on discrete series*, *Prog. Theor. Phys.* **107** (2002) 1245 [hep-th/0107189];
A. Rajaraman and M. Rozali, *Boundary states for D-branes in AdS_3*, *Phys. Rev.* **D 66** (2002) 026006 [hep-th/0108001];
A. Parnachev and D.A. Sahakyan, *Some remarks on D-branes in AdS_3*, *JHEP* **10** (2001) 022 [hep-th/0109150];
P. Lee, H. Ooguri and J.W. Park, *Boundary states for AdS_2 branes in AdS_3*, *Nucl. Phys.* **B 632** (2002) 283 [hep-th/0112188];
B. Ponsot, V. Schomerus and J. Teschner, *Branes in the euclidean AdS_3*, *JHEP* **02** (2002) 016 [hep-th/0112198];
D. Israel, *D-branes in lorentzian AdS_3*, *JHEP* **06** (2005) 008 [hep-th/0502159];
Y. Hikida, R.R. Nayak and K.L. Panigrahi, *D-branes in a big bang/big crunch universe: Nappi-Witten gauged WZW model*, *JHEP* **05** (2005) 018 [hep-th/0503148].

[5] A. Sen, *Rolling tachyon*, *JHEP* **04** (2002) 048 [hep-th/0203211].

[6] A. Sen, *Tachyon matter*, *JHEP* **07** (2002) 065 [hep-th/0203265].

[7] A. Sen, *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[8] A. Sen, *Tachyon dynamics in open string theory*, *Int. J. Mod. Phys.* **A 20** (2005) 5513 [hep-th/0410103].

[9] D. Kutasov, *D-brane dynamics near NS5-branes*, hep-th/0405058.

[10] D. Kutasov, *A geometric interpretation of the open string tachyon*, hep-th/0408073.

[11] Y. Nakayama, Y. Sugawara and H. Takayanagi, *Boundary states for the rolling D-branes in NS5 background*, *JHEP* **07** (2004) 020 [hep-th/0406173];
B. Chen, M. Li and B. Sun, *Dbrane near NS5-branes: with electromagnetic field*, *JHEP* **12** (2004) 057 [hep-th/0412022];
Y. Nakayama, K.L. Panigrahi, S.J. Rey and H. Takayanagi, *Rolling down the throat in ns5-brane background: the case of electrified D-brane*, *JHEP* **01** (2005) 052 [hep-th/0412038].

[12] D.A. Sahakyan, *Comments on D-brane dynamics near NS5-branes*, *JHEP* **10** (2004) 008 [hep-th/0408070];
H. Yavartanoo, *Cosmological solution from D-brane motion in NS5-branes background*, hep-th/0407079;
K.L. Panigrahi, *D-brane dynamics in Dp-brane background*, *Phys. Lett.* **B 601** (2004) 64 [hep-th/0407134];
A. Ghodsi and A.E. Mosaffa, *D-brane dynamics in RR deformation of ns5-branes background and tachyon cosmology*, *Nucl. Phys.* **B 714** (2005) 30 [hep-th/0408015];
J. Kluson, *Non-BPS D-brane near NS5-branes*, *JHEP* **11** (2004) 013 [hep-th/0409298]; *Non-BPS Dp-brane in the background of NS5-branes on transverse $\mathbb{R}^3 \times S^1$*, *JHEP* **03** (2005) 032 [hep-th/0411014]; *Non-BPS Dp-brane in Dk-brane background*, *JHEP* **03** (2005) 044 [hep-th/0501010]; *Note about non-BPS and BPS Dp-branes in near horizon region of N*

Dk-branes, *JHEP* **03** (2005) 071 [hep-th/0502079]; J. Kluson, *Remark about non-BPS Dp-brane at the tachyon vacuum moving in curved background*, hep-th/0504062;
O. Saremi, L. Kofman and A.W. Peet, *Folding branes*, *Phys. Rev.* **D 71** (2005) 126004 [hep-th/0409092];
D. Bak, S.J. Rey and H.U. Yee, *Exactly soluble dynamics of (p,q) string near macroscopic fundamental strings*, *JHEP* **12** (2004) 008 [hep-th/0411099];
S. Thomas and J. Ward, *D-brane dynamics and NS5 rings*, *JHEP* **02** (2005) 015 [hep-th/0411130]; *D-brane dynamics near compactified NS5-branes*, *JHEP* **06** (2005) 062 [hep-th/0501192]; *Geometrical tachyon kinks and NS5 branes*, *JHEP* **10** (2005) 098 [hep-th/0502228]; *Inflation from geometrical tachyons*, *Phys. Rev.* **D 72** (2005) 083519 [hep-th/0504226]; *Fuzzy sphere dynamics and non-abelian DBI in curved backgrounds*, hep-th/0508085;
B. Chen and B. Sun, *Note on DBI dynamics of D-brane near NS5-branes*, *Phys. Rev.* **D 72** (2005) 046005 [hep-th/0501176];
W.H. Huang, *Tubular solutions in ns5-brane, Dp-brane and macroscopic strings background*, *JHEP* **02** (2005) 061 [hep-th/0502023];
J. Kluson and K.L. Panigrahi, *Supertube dynamics in diverse backgrounds*, *JHEP* **08** (2005) 033 [hep-th/0506012];
Y. Nakayama, S.-J. Rey and Y. Sugawara, *D-brane propagation in two-dimensional black hole geometries*, *JHEP* **09** (2005) 020 [hep-th/0507040].

[13] W.-H. Huang, *Anti-de Sitter D-branes in curved backgrounds*, *JHEP* **07** (2005) 031 [hep-th/0504013].